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A PARETO OPTIMAL GROUP DECISION PROCESS

By

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Introduction

Despite the fact that men have been making collective decisions for thousands of years, suggestions of new technologies for making these decisions are rare. This intellectual inactivity is apparently borne of frustration, not of complacency. The aversive predicament is that while a generally efficient decision process must be based upon true individual evaluations of group alternatives, any kind of decision process based upon the free market, democratic voting, or authoritarian rule cannot generally induce individuals to reveal the true strength of their evaluations.

This long-standing predicament has been concisely explained by Professors Samuelson and Musgrave in their insightful modernizations of Wicksell's theory of public finance. (1) As regards decisions guided by the free market, it is in the self-interest of each individual to grossly understate the true value which he places on a collective alternative because he will receive close to the same benefit regardless of the amount which he states he is willing to contribute. As regards democratic voting, any voter is physically incapable of expressing the amount of wealth that he is willing to surrender in order to obtain the outcome which he desires. Even if the ballot requested such information, the voter would have no incentive to tell the truth. Finally, central authorities find that individual evaluations of group alternatives are highly variable and unobservable in practice.

Nevertheless, the free market, democratic voting and authoritarian rule do not logically exhaust the possible group decision processes. There may be other feasible group decision processes.

In this note a new type of group decision process is put forth. The process is based upon a scheme for insuring each individual against political misfortune. Under specifications stated below, an individual's insurance purchases reveal his true evaluations of collective alternatives. The Wicksell-Samuelson-Musgrave predicament is side-stepped because non-paying individuals are completely excluded from enjoying the benefits of insurance even though individuals who would refuse to pay for the insured alternative would not be excluded from enjoying the benefits of that alternative.

We shall prove a theorem that this group decision process, by exploiting the special properties of individual insurance decisions, leads to a Pareto optimal allocation of resources.

(A Pareto optimal allocation of resources, or a "Pareto optimum," is an allocation of resources for which there is no reallocation which can make everyone better off. At a Pareto optimum, if someone is made better off, someone else must be made worse off. For each of the various attainable distributions of utility, there is a different Pareto optimal allocation. The desirability of the actual choice among Pareto optimal allocations is metaphysically determined as a matter of distributional equity. A Pareto optimum which is also an optimum from the standpoint of distributional equity is called a <u>complete</u> optimum.)

Except for the supporting institutions, our theorem is not restricted to any special physical environment. For example, there may or may not be social or technological interaction, political institutions, individual ownership and exchange, pure collective goods, or uncertainty. In addition, the theorem is not restricted to any particular social group; it applies as well to a family as it does to a civilization. The formal study of choice in this very general kind of environment has been pioneered by Professor K. J. Arrow. (2) His remarkable result, the General Possibility Theorem, states that it is impossible to construct a group decision process which generally yields a <u>complete</u> optimum based upon individual values. Nevertheless, it <u>is</u> possible, at least conceptually, to construct a group decision process which generally yields a Pareto optimum in this unrestricted environment. The obstacle in the way of the actual construction of such a process has been the conventional Wicksell-Samuelson-Musgrave predicament. Our theorem will show that this obstacle can be hurdled through the implementation of the process described below.

<u>The Group Decision Process</u>

In an iteration of our decision process the government (group administrator) initially presents the citizenry (group members) with a collective choice between a pair of alternatives and then immediately offers to insure each individual against the occurrence of either alternative. The group decision rule is to choose that alternative for which the payoff on claims will fall short of the receipts from the collection of insurance premiums. In other words, the government selects the alternative which will yield a surplus to the insurance operation. Ties are decided in favor of the status quo. Having eliminated one alternative, the government goes on to consider other, uneliminated, politically feasible alternatives until no feasible alternative can win against an existing status quo. The entire decision process will be called the D-Process.

We assume that there is no more than a countable infinity of feasible alternatives so that the process, by eliminating one alternative on each iteration, must approach some solution. The final surplus is returned via lump-sum subsidies, payments to be made independently of individual insurance behavior and actual group decisions.

There are generally several possible solutions to the D-Process, each corresponding to different possible orders in which the government may present the alternatives. Nevertheless, under specifications stated below, all possible solutions will be proved to be Pareto optimal.

Definitions and Notation

Let there be N people ($k = 1, \ldots, N$). Let the alternatives relevant to the current iteration be A and $\tilde{A}$. $\tilde{A}$ is the existing allocation of resources, the status quo. Individual k evaluates A to be V_k dollars more attractive than $\tilde{A}$ (V_k may be positive, zero, or negative). $\tilde{A}$ is a Pareto optimum if there is no feasible A for which $\sum_{k=1}^{N} V_k > 0$. Such an $\tilde{A}$ is a Pareto optimum by the definition of such an optimum. For since there is no change from this $\tilde{A}$ for which the gainers are willing to over-compensate the losers; there is no change which can make everyone better off. (3)

Let P_k be the prior probability which individual k places on the occurrence of (victory for) $\tilde{A}$. $(1-P_k)V_k$ is then the expected value of the prospect of the social change to individual k.

The government-insurance company sets the odds against victory for $\tilde{A}$ at $(\frac{1}{P_G} - 1):1$. P_G is their implicit estimate of the probability that $\tilde{A}$ will occur. Thus, if individual i is insuring himself against the defeat of A by paying out X_i on insurance premiums, he receives $(\frac{X_i}{P_G} - X_i)$ if A loses and $(-X_i)$ if A wins. Similarly, if individual j is insuring against a victory for A by paying out Y_j on insurance premiums, he receives $(-Y_j)$ if A loses and $(\frac{Y_j}{1 - P_G} - Y_j)$ if A wins. Let the first n people be those who insure against the defeat of A and the remaining N - n people be those who insure against a victory for A. Government receipts from the collection of insurance premiums are

$$\sum_{i=1}^{n} X_i + \sum_{j=n+1}^{N} Y_j.$$

If A loses, the total value of the insurance claims against the government is

$$\frac{1}{P_G} \sum_{i=1}^{n} X_i;$$

if A wins, these claims amount to

$$\frac{1}{1 - P_G} \sum_{j=n+1}^{N} Y_j.$$

Notice that

$$\frac{1}{P_G} \sum_{i=1}^{n} X_i > \sum_{i=1}^{n} X_i + \sum_{j=n+1}^{N} Y_j$$

if and only if

$$\frac{1}{1 - P_G} \sum_{j=n+1}^{n} Y_j < \sum_{i=1}^{n} X_i + \sum_{j=n+1}^{N} Y_j.$$

Hence, ruling out ties, there is one and only one of the two current alternatives which can yield a government surplus. The decision rule is unambiguous.

Thus, given the two group alternatives relevant to the current iteration, A and $\widetilde{A}$, his own prior probability P_k, and the insurance offer, $(\frac{1}{P_G} - 1):1$, each individual is guided by his preferences to spend a certain amount on insurance, X or Y.

The Specifications

So far we have only described the D-Process. We have yet to specify how the various probabilities are determined, how individual insurance decisions are related to individual evaluations, and how individual evaluations are affected by social decisions. Once these specifications are made we can relate individual evaluations to social decisions so as to judge the efficiency of the process. Three rather restrictive, but conventional, specifications will be made. It will then be proved that they allow the D-Process to rest only at Pareto optimal allocations.

First, we assume that the government (or some other institution) is the only source of information as to the likelihood of each outcome of the election. The government then determines, by announcement, the P_k's. P_G is set equal to the same number. Hence, for all k, $P_k = P_G$. This probability is assumed to be greater than zero and less than one. Since people are not really completely in the dark concerning the outcome, the probability announced by the government would have to be regarded as far superior to the estimates of the individuals. This superiority might be established through the government's use of sampling devices, polls, propaganda, or appropriate taxes on the behavior of those bold individuals who indicate some consistent pattern of disagreement with the government probability estimates. For any one decision, the probability announced by the government need not -- no, cannot -- be related to any "true" probability. This is because the D-Process, the method of generating the actual outcomes, is not a random mechanism. There is only a lack of information concerning the eventual outcome, and this ignorance is summarized in the prior probability. The irrelevance of the numerical magnitude of this probability is apparent from Eq. (3) in the proof of the theorem below.

Second, we assume that all individuals are risk-aversive. For our purposes, this means that all individuals marginally prefer any certain amount of wealth in the hand to an uncertain prospect of equivalent expected value. Although it does leave much to be desired, this specification is probably true for most people, especially when the stakes are high. The gamblers, like the outguessers who violate the previous specification, should be somehow identified and taxed according to their particular abnormality. Specifying practical methods of identifying such individuals is no mean task and will not be attempted here.

Third, we assume that each individual's marginal rate of substitution between each government service and income is independent of his income level. This prevents the V_k's from changing during the working out of the D-Process. This assumption (which sometimes goes by the "constant marginal utility of income" misnomer) will be called the indexability of public goods into money. Since the assumption permits an individual to rank the alternatives independently of his income (which is determined by the alternatives, probabilities, and surplus distribution policies), each individual can quantitatively rank the alternatives in terms of money values. Thus, the sum of these values over all individuals for each alternative can be given a unique, finite, numerical value, and there must be at least one alternative for which there are no alternatives with a higher value. In summary, as a result of this assumption, Pareto optimal allocations exist and are defined independently of the decision process. (4)

The non-stochastic nature of our first two specifications makes them appear overly restrictive. In fact, while the influence of the gamblers and the outguessers will probably prevent the exact achievement of any optimum, the noises which they create are not systematic and should be relatively uninfluential when important or controversial issues are at stake. Furthermore, these specifications are conventional in that they parallel conditions which are required in general demonstrations of the Pareto optimality of the process of competitive production and exchange. (5) Unlike the D-Process, this free-market-process requires a heavy load of additional assumptions concerning social and technological interdependence, political institutions, individual ownership and exchange, collective goods, and market stability.

Theorem

If $0 < P_k = P_G < 1$, everyone is risk-aversive, and public goods are indexable into money, then any final allocation of resources resulting from the D-Process is a Pareto optimum.

Proof: The expected value of the social change to individual k is

$$(1-P_k)V_k = (1-P_G)V_k.$$

The expected value of the government insurance policy is

$$P_G\left(\frac{1}{P_G} - 1\right)X_k + (1 - P_G)X_k = 0.$$

The insurance policy has no effect upon k's expected income.

Individual i, whose V_i is positive, converts his possible payoffs (wealth increments) from $(V_i, 0)$ to $(V_i - X_i, \frac{X_i}{P_G} - X_i)$. Since he is risk-aversive,

$$V_i - X_i = \frac{X_i}{P_G} - X \text{ , or}$$

(1) $$P_G V_i = X_i .$$

For individual j, whose V_j is negative, $(V_j, 0)$ is changed to

$$(V_j + \frac{Y_j}{1-P_G} - Y_j, \ -Y_j),$$

where

$$V_j + \frac{Y_j}{1-P_G} - Y_j = -Y_j \text{, or}$$

(2) $$(P_G - 1)V_j = Y_j .$$

From the definition of the D-Process and using Eq. (1) and (2), the government chooses A if and only if

$$\sum_{i=1}^{n} X_i + \sum_{j=n+1}^{N} Y_j - \frac{1}{P_G - 1} \sum_{j=n+1}^{N} Y_j > 0,$$

$$P_G \sum_{i=1}^{n} V_i + (P_G - 1) \sum_{j=n+1}^{N} V_j + \sum_{j=n+1}^{N} V_j > 0,$$

(3) $$\sum_{k=1}^{N} V_k > 0.$$

When $\tilde{A}$ is Pareto optimal, no A exists which satisfies Eq. (3), and the process rests at $\tilde{A}$. If $\tilde{A}$ is not Pareto optimal, there could not have been a Pareto optimal alternative in previous iterations, for the indexability of money into public goods assures us that Eq. (3) would have held and $\tilde{A}$ would not have survived. And since there is at least one Pareto optimum among the feasible alternatives, $\tilde{A}$ cannot survive future iterations. Hence the final allocation is nothing other than a Pareto optimum.

Q. E. D.

1. See R. A. Musgrave, The Theory of Public Finance (New York: McGraw-Hill, 1959), Chapters IV, VI. Also, Paul A. Samuelson, "The Pure Theory of Public Expenditure," Review of Economics and Statistics, XXXVI (November 1954), pp. 387-389.

2. K. J. Arrow, Social Choice and Individual Values, 2nd edition, (New York: Wiley and Sons, 1963), especially Chapters IV and V.

3. Also see Arrow, op. cit., pp. 34-37, 96.

4. When the V_k's vary with income, we must recognize that valuations will depend upon probabilities, expected surpluses, and the history of the decision process. A Pareto optimal A can be rejected because of faulty estimates of wealth during the iteration in which the A is considered. However, by allowing every alternative to be considered and by allowing P_G to approach zero before any A is rejected, the D-Process and the theorem below can be generalized to cases of variable V_k's.

5. The necessity of a general form of the first assumption for the unambiguous optimality of the competitive process is a central theme of F. Knight's, Risk, Uncertainty and Profit. Different opinions about the true probability distributions controlling future events would lead a free-market to ownership patterns which either waste real resources through the negotiation and enforcement of superfluous leasing contracts or place the control of real assets in the hands of relatively inefficient speculators. (In the language of economic theory, "uncertainty" leads to "imperfect capital markets.")

The necessity of the second assumption was proved in K. J. Arrow's "The Role of Securities in the Optimal Allocation of Risk-Bearing," Review of Economic Studies, April, 1964. With some people preferring risk, there may easily be no market odds capable of generating competitive equilibrium in a market for conditional claims (e.g., private insurance policies).