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Characteristics of Worlds with Perfect Strategic Communication*

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A new model of rational, two-person interaction is implicit, especially in the numerical examples of Chapter V, in Thomas Schelling's classic, "The Strategy of Conflict" [6]. In the implicit model, one of the two players first commits himself to a reaction function and then the second player selects his rational action given the first's prior reaction function. The prior committed reaction function is that function which maximizes the first player's payoff given the known, rational response of the second player. Yet somehow, this implied model—which employs substantially more information than the standard two-person von Neumann–Morgenstern "perfect information" game in that one player is allowed to communicate his strategy to the other before the other selects his strategy—has been largely neglected in subsequent theoretical analyses.[1]

* The current paper is a further development and generalization of the author's contributions to an unpublished, 1974–1975, joint paper with Roger Faith entitled, "A Theory of Games with Truly Perfect Information."

[1] This is perhaps because Professor Schelling did not properly contrast his implied game-theoretic model with more conventional games. In particular, he did not see that he was merely applying the Zermelo–von Neumann–Morgenstern perfect information solution concept to strategies rather than actions (or "plays of a game"). While von Neumann and Morgenstern explicitly recognized [8, Sect. 11.3] that games could be constructed in which strategies are communicated in the same way as the actions in their perfect information games, they saw nothing novel about such games, for they pose no new problem in the development of solution concepts or the existence of solutions. Perhaps, had they been more interested in evaluating the Pareto optimality of solution actions or in formally capturing the microeconomics of social institutions, they would have devoted more intellectual resources to games with perfect information concerning strategies as well as actions. But von Neumann and Morgenstern also expressed serious doubts about having players rely on the rationality of others, a reliance required by their perfect information solution concept. Their argument

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The present paper can be viewed as an attempt to remedy this neglect. In particular, the paper will: (1) Derive Schelling's implicit model from a prior model containing perfect information regarding the strategies of others and simultaneously generalize the model to n-players; (2) establish the existence and Pareto optimality of a solution allocation resulting from the general model given a finite set of possible outcomes and only strict preferences between possible outcomes; (3) show that indifference between possible outcomes allows Pareto nonoptima to enter the solution set under a certain condition; (4) show how the model is distinct from other game-theoretic models in that it contains a theory of individually rational communication and, correspondingly, a theory of individually rational "cooperation";[2] and (5) identify conditions under which the model's basic optimality characteristics hold up under a generalization admitting an endogenous determination of the order of strategy selection.

The following sections of this paper correspond, respectively, to these five extensions of Schelling's work. In addition, regarding possible empirical applications, Section III outlines a possible explanation for the appearance and disappearance of slavery while Section V outlines a possible explanation for the tendency toward efficiency in observed social institutions.

I. The Model and Its Solutions

A. *The Physical Environment and Institutional Possibilities*

An individual is denoted i, $i = 1,\dots, n$. An action of individual i is denoted x_i, where $x_i \in X_i$, a finite set of feasible actions of individual i. A possible social choice, or *allocation*, is defined by an n-dimensional set of actions, and is denoted $x = (x_1, x_2, \dots, x_n)$, so that $x \in \prod_{i=i}^{n} X_i$. To describe

supporting these doubts [8, Sect. 4.1.2] is that it may pay a player to deviate from "rational" responses if he knows that another player's strategy depends on his responses. But it is precisely these deviations that are at the heart of any theory of strategic communication, a theory that allows some players to make rational deviations out of a set of possible deviations from what otherwise would be their future rational choices. For example, since the last deliverer in a transaction always has an incentive to withold delivery, he must devise and communicate a commitment to deviate in a specified way from his narrowly rational last response in order to induce prior deliveries by others. Von Neumann and Morgenstern did not see that their justifiable skepticism with respect to their "perfect information" game leads toward the adoption of games with *more* information than appears in their "perfect information" game rather than towards the imperfect information games which they so elegantly explored.

[2] This is not to say that conventional cooperative game theory cannot be reformulated to produce effects which are similar to those characterizing our generalization of Schelling's model. Indeed, such a reformulation has been recently achieved by Rosenthal [5]. Section IV will indicate, however, that the basic assumptions of cooperative game theory are inconsistent with perfect strategic communication.

individual preferences, each individual, i, is given a complete, transitive, irreflexive, antisymmetric, binary relation, $>_i$, defined over $\prod_{i=1}^{n} X_i$. This description rules out indifference between elements of the finite set of possible allocations. The motivation for this assumption and the effects of indifference on our central results will be discussed later. A Pareto optimum is an allocation, $x', x' \in \prod_{i=1}^{n} X_i$, for which there is no alternative allocation, $x'', x'' \in \prod_{i=1}^{n} X_i$, such that $x'' >_i x'$ for all i. Several Pareto optima may exist.

The institutional constraints on individual actions, i.e., the reactions of other to his actions, are not taken here as given; they are derived. When there is perfect strategic communication, this is done by allowing individuals to sequentially communicate their respective reaction functions. Thus, for perfect information regarding institutions, the first individual to establish and communicate a reaction function, labeled individual 1, presents the reaction function, $x_1 = f_1(x_2, \ldots, x_n)$, to the other individuals; the second communicator, labeled individual 2, then presents $x_2 = f_2(x_3, \ldots, x_n)$ to individuals 3 through n; and so on up to the $n-1$st communicator, who presents $x_{n-1} = f_{n-1}(x_n)$ to the nth individual, who has no need to communicate. Once the action of the nth individual is taken, the action of the $n-1$st individual is determined. Once this pair of actions is taken, the action of individual $n-3$ is determined, and so on up until an allocation is determined as a chain reaction from the nth individual's action. The set $(f_1, f_2, \ldots, f_{n-1})$ is thus a complete institutional description. The feasible choice set, or strategy set, of individual 1 is the set of *all* functions from $\prod_{i=2}^{n} X_i$ to X_1. This can be represented by the functional variable, F_1. Similarly, $F_2, \ldots, F_{n-1}$ can be used to represent the strategy sets of individuals 2 to $n-1$. The product space, $\prod_{i=1}^{n-1} F_i$, thus represents the world's institutional possibilities. Denoting n's strategy set, X_n, by F_n, and his choice, x_n, by f_n, $F = (F_1, \ldots, F_n)$ denotes the social strategy space and $f = (f_1, \ldots, f_n)$ a particular outcome.[3]

[3] A question may arise as to why some individuals do not present reaction functions to other individuals who are higher up in the communication hierarchy. Consider individual n. Facing the prior strategies of the other $n-1$ individuals, he sees that the eventual allocation must be consistent with the chosen reaction functions of each of the $n-1$ prior selectors. Hence, if individual n responds to the prior selectors with a simple action, he will have a free choice over all allocations consistent with the prior reaction functions. But if n responds with a function of prior actions, thus giving further choices to the prior strategy selectors, he can only reduce his original choice out of the same set of possible allocations. He cannot expand the set of possible outcomes because any eventual outcome must be consistent with the given $n-1$ reaction functions. Similarly, if the $n-1$st strategy selector presents a reaction function rather than an action to his prior strategy selectors for a given action of individual n, he is giving them the choice of actions consistent with the set of reaction functions he faces and thus can be no better off. This also applies, in like fashion, to individuals $n-2$ to 2, so that it is in no individual's interest to present a reaction function to a prior strategy selector.

While Howard [2] has considered a game containing strategies contingent on the strategies of subsequent strategy selectors, he adopts a Nash solution concept [3], wherein the first strategy selector accepts *as given* the strategies of subsequent strategy selectors. This is, of course, generally irrational under perfect strategic communication. For the choice of the first strategy selector may obviously affect the choice of the subsequent strategy selectors. The solution concept appropriate to perfect strategic communication is a von Neumann–Morgenstern perfect information—not a Nash—solution concept.

B. *Equilibrium Institutions, or "Solutions"*

A solution, $(f_1^*,...,f_n^*)$, is a set in which the ith variable is, for each i, maximal with respect to $>_i$ for given values of $f_1,...,f_{i-1}$. A solution can be constructed as follows: First, we find, for individual n, f_n^*, the point in F_n such that, for all $f_n \neq f_n^*, f_n \in F_n$,

$$\{f_1(f_2,...,f_{n-1},f_n^*), f_2(f_3,...,f_n^*),...,f_n^*\} \\ >_n \{f_1(f_2,...,f_{n-1},f_n), f_2(f_3,...,f_n),...,f_n\}.$$

This solution determines a dependency of f_n^* on $f_1, f_2,...,$ and f_{n-1}, which we write $f_n^*[f_1,...,f_{n-1}]$. Then, for individual $n-1$, we find a reaction function in F_{n-1}, f_{n-1}^*, such that for all $f_{n-1} \in F_{n-1}, f_{n-1} \neq f_{n-1}^*$,

$$\{f_1(f_2,...,f_{n-2},f_{n-1}^*,f_n^*[f_1,...,f_{n-2},f_{n-1}^*]),..., \\ f_{n-2},f_{n-1}^*,f_n^*[f_1,...,f_{n-2},f_{n-1}^*]\} \\ >_{n-1} \{f_1(f_2,...,f_{n-2},f_{n-1},f_n^*[f_1,...,f_{n-2},f_{n-1}]),..., \\ f_{n-2},f_{n-1},f_n^*[f_1,...,f_{n-2},f_{n-1}]\}.$$

This solution determines the dependency of f_{n-1}^* on $f_1, f_2,...,$ and f_{n-2}, which we describe as $f_{n-1}^*[f_1,...,f_{n-2}]$. Then, for individual $n-2$, we find a reaction function, f_{n-2}^*, such that, for all $f_{n-2} \in F_{n-2}, f_{n-2} \neq f_{n-2}^*$,

$$\{f_1(f_2,...,f_{n-2}^*,f_{n-1}^*[f_1,...,f_{n-2}^*],f_n^*[f_1,...,f_{n-2}^*,f_{n-1}^*[f_1,...,f_{n-2}^*]]),..., \\ f_{n-2}^*,f_{n-1}^*[f_1,...,f_{n-2}^*],f_n^*[f_1,...,f_{n-2}^*,f_{n-1}^*[f_1,...,f_{n-2}^*]])\} \\ >_{n-2} \{f_1(f_2,...,f_{n-2},f_{n-1}^*[f_1,...,f_{n-2}],f_n^*[f_1,...,f_{n-2},f_{n-1}^*[f_1,...,f_{n-2}]]),..., \\ f_{n-2},f_{n-1}^*[f_1,...,f_{n-2}],f_n^*[f_1,...,f_{n-2},f_{n-1}^*[f_1,...,f_{n-2}]])\}.$$

This solution thus determines the dependency of f_{n-2}^* on $f_1, f_2,...,$ and f_{n-3}, which we write as $f_{n-2}^*[f_1,...,f_{n-3}]$. The process continues until we have determined f_1^*. Since f_1^* does not depend on any prior functions, we can use it to determine the succeeding reaction functions by successively substituting starred values into $f_2^*[f_1], f_3^*[f_1,f_2],...,$ and $f_{n-1}^*[f_1,f_2,...,f_{n-2}]$.

In this way, a solution, $f^* = (f_1^*, f_2^*, \ldots, f_n^*)$, which implies a solution allocation, $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$, is determined.

The finite structure of the successive maximization problems, along with the completeness and transitivity of $>_i$, assures us that a solution *always* exists.

II. Pareto Optimality

The institutions formed under perfect strategic communication induce Pareto optimal allocations. To prove this, suppose the solution, x^*, is not Pareto optimal. Then there is a point, $x^0 \in \prod_{i=1}^n X_i$ such that $x^0 >_i x^*$ for all i. A set of reaction functions generating x^0 as an allocation is given by $(f_1^0, \ldots, f_{n-1}^0)$. Of course $(f_1^*, \ldots, f_{n-1}^*) \neq (f_1^0, \ldots, f_{n-1}^0)$; otherwise, x^0 would be the solution. Now let individual 1 consider:

$$\begin{aligned} f_1(f_2, \ldots, f_n) &= f_1^0 \qquad \text{if } (f_2, \ldots, f_n) = (f_2^0, \ldots, f_n^0) \\ &= f_1^* \qquad \text{otherwise.} \end{aligned} \tag{A}$$

This may induce each subsequent strategy selector to reorder his strategy in $(f_2^0, \ldots, f_n^0)$ relative to $(f_2^*, \ldots, f_n^*)$. However, as it does not alter the allocations resulting from nonsolution strategies other than $(f_2^0, \ldots, f_n^0)$, it does not alter anyone's ordering of these other strategies relative to $(f_2^*, \ldots, f_n^*)$. Therefore, because $x_0 >_1 x^*$, individual 1 is no worse off under (A) than under his original strategy.

We next let individual 2 consider, in view of (A),

$$\begin{aligned} f_2(f_3, \ldots, f_n) &= f_2^0 \qquad \text{if } (f_3, \ldots, f_n) = (f_3^0, \ldots, f_n^0) \\ &= f_2^* \qquad \text{otherwise.} \end{aligned} \tag{B}$$

This similarly cannot hurt individual 2. We continue on to individual n, who now faces (A), (B),.... Thus, $(f_1^0, \ldots, f_n^0) = x^0$ will result if he picks $x_n = x_n^0$; and $(f_1^*, \ldots, f_n^*)$ if he picks his solution action. Since $x^0 >_n x^*$, he picks the former. The supposition that there is a Pareto nonoptimal solution is thus immediately contradicted: For the supposition implies that the players individually prefer a nonsolution set of strategies, $(f_1^0, \ldots, f_n^0)$, to the solution set, $(f_1^*, \ldots, f_n^*)$.

III. The Difficulties Presented by Indifference

If someone other than the first strategy selector were indifferent between two or more possible solution strategies, a prior selector, who would otherwise have no way of knowing what the indifferent one would do, would—assuming that he could perform a reaction that would leave this later selector uniformly worse off than in a solution—simply adjust his reactions to all but one of the later-selected strategies so as to make these strategies suboptimal for the later selector. The resulting solution in this case is also Pareto optimal, as can be seen by noting that the above optimality proof also applies here as long as the Pareto dominating strategy used in the proof is still feasible, which is the case because no prior strategy selector, in inducing a specific choice of a later selector between strategies about which the later selector would otherwise be indifferent, would eliminate the Pareto superior strategy choice.

However, when the *first* strategy selector is indifferent, Pareto nonoptima may easily arise. Consider the following "slave master's insensitivity" payoff matrix:

Master \ Slave	Work x_2^1	Rest x_2^2
Beat the slave x_1^1	5, –10	0, –6
Insult the slave x_1^2	10, –4	1, 0
Leave the slave alone x_1^3	10, 0	0, 4

(The standard, VNM–Nash, no-regret solution has the slave resting while the master insults the slave; this is both nonoptimal *and* empirically unrealistic!) The solution set under perfect strategic communication, with the master as the first strategy selector, contains the Pareto optimum (10, 0), where the master will beat the slave if he rests and leave him alone if he works. But the set also contains (10, –4), as the master may also insult the slave, lowering the slave's benefit to –4 without altering either the master's payoff or the slave's optimal decision. The point, (10, –4), is obviously Pareto inferior to (10, 0).

The exercise of Section II can be repeated for a weak preference relation to show that if x^*, a Pareto nonoptimum, is a solution, so is x^0. Hence, although the solution set with weak preference relations may sometimes contain a Pareto nonoptimal point, it must always also contain a Pareto optimum.

Because all standard *competitive* equilibria are Pareto optimal, economists have grown accustomed to the thought that individual indifference between various possible equilibria is unimportant. But, as individual indifference between the possible equilibria of a master–slave relationship can induce Pareto nonoptima, we should guard against the habit of ignoring solution indifference when examining decentralized slave economies. Apparently, the real world has not ignored the problem. As our model would predict, observed decentralized slavery systems have arisen only through the capture of social "outsiders" toward which initial benevolence could hardly have been widespread among insiders [1] and have dissolved not by slave uprisings or voluntary manumissions but, at least in modern times, by the intervention of politically powerful humanitarians armed with a "moral argument" [1] based on examples in which slaves were torn from their families, worked to death, tortured, or broken of spirit for the minor conveniences of their only mildly benevolent, and therefore largely indifferent, masters.[4]

IV. Contrast with Cooperative Game Theory

The commitment of our players to their communicated strategies simultaneously prevents them from forming the blocking coalitions with subsequent strategy selectors that they would under the narrowly rational decision structure of standard cooperative game theory (e.g., [4]). It is such narrowly rational decision-making, or implicitly imperfect strategic communication, that is responsible for the generally unsatisfactory, overly full or empty, solution sets in standard cooperative game theory.

Consider, for example, a "majority game," a three-person, zero-sum, game in which, say, a dime and a nickel are to be shared by the three players. If players 1 and 2 each select certain actions implying that they "get together," 2 gets a dime and 1 gets a nickel. If 1 and 3 each select certain actions, where the action is different for 1 than in the former case, then 1 gets a dime and 3 gets a nickel. If 2 and 3 each select new actions implying that they "get together," then 3 gets a dime and 2 gets a nickel. Cooperative game theory offers no meaningful solution to this game because, for any distribution of coins, there is a blocking coalition. Under perfect strategic communication, where the order of strategic selection is, say, 1, 2, 3, player 1 will adopt the following strategy: "I will get together with 2 if he gets together with me; otherwise, I will perform my part of getting together with 3." Player 2 then selects: "I will perform my part of getting together with 1 regardless of the action of player 3." Player 3 gets nothing no matter what

[4] While abolition has sometimes also served to redistribute away from the masters, as it apparently did in the American South in view of the slow pace of Southern Reconstruction, in most cases freed slaves become serfs or debt-peons who provide about the same benefit as do slaves to the capitalist class [1]. The social advantage of serfdom and debt-peonage is that they prevent local slave master's insensitivity problems, the former having central authorities rigidly controlling the taxation of the immobile serfs and the latter granting a choice of creditor-employers to the peon.

he does. It is easy to verify that there is no other solution. In sharp contrast, under standard cooperative game theory, 3 would offer to get together with 1, who—being unable to commit himself to a fixed response—would be unable to refuse the offer. And we would be off on the never-ending cycle of coalition formation characteristic of standard cooperative game theory.

V. Determination of Hierarchical Positions and Empirical Application

While one may think of the order of strategy selection in the above model as being arbitrarily determined by the "rules of the game," or by "initial endowments," it is much more realistic to determine the order of strategy selection in a higher-order game. When the higher-order game contains an outside player who assigns hierarchial positions through his ability to punish inside players, the outsiders may alter the above solution by also constraining the forms of insider reaction functions, essentially imposing his own prior, not-necessarily-rational, reaction function. The model of this paper need not apply in this case. A model attempting a realistic description of such prior constraints, and a demonstration of the simultaneous existence of an equilibrium order of strategy selection and a set of constrained strategies under noncompetitive interdependence, is developed elsewhere [7]. This study finds that the constrained reaction function equilibrium is still approximately Pareto optimal. This optimality result, when complemented by numerous others of this author and associates indicating the approximate Pareto optimality of observed government policy towards *competitive* sectors with realistic externalities, indicates that a powerful force toward Pareto optimality underlies observed institutions.

Our model would *predict* this optimality result if we could assume that: (1) the outside players imposed rational constraints *and* (2) a prior, hierarchy-determining higher-order game containing no outside players had no effect on the optimality properties of the lower-order game. The second assumption, as well as the first, is plausible, for while the anarchistic battle for hierarchical position that occurs in thee absence of an umpire or outside enforcer is a warlike, generally Pareto inefficient, Nash–VNM noncooperative struggle to establish prior commitments, war losses are strictly sunk costs one a hierarchy is formed and our own, lower-order game is ready to be played. Hence, once the natural, unavoidable, dead-weight losses in establishing a hierarchy have been incurred, our model can be applied. Its central optimality results then serve to explain the observed efficiency of actual social institutions.

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